# Dynamic Behavior of an Aircraft Encountering Aircraft Wake Turbulence

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**This paper deals with the dynamic behavior of an airplane encountering aircraft wake turbulence. A digital computer simulation was developed to study the response of an aircraft flying into a trailing vortex wake. The simulation includes the complete six degree-of-freedom equations of motion, a description of the vortex velocity field, unsteady aerodynamics, and pilot control input. The parameters, varied in this simulation, include the penetration angle, separation distance, aircraft size (for both the penetrating and generating aircraft), and pilot control input (single-or multi-axes). Predicted vortex induced motions are presented for several probe aircraft. The probe aircraft selected are representative of general aviation, business, and light jet transport type aircraft, whereas the aircraft used to generate the vortex wakes are representative of the commercial transport fleet. The computer predictions indicate that relatively large aircraft (80,000 lb jet transport) can experience unacceptable vortex induced roll excursions. In addition to evaluating the vortex induced responses, the effect of pilot control input was assessed. It was found that pilot control input could be momentarily out of phase with the vortex induced disturbance. Thus, in certain cases, the pilot's control input would tend to aggravate the vortex-induced upset.**

## Nomenclature

$a$ = vortex core radius
$b$ = wing span
$C_l$ = rolling moment coefficient - rolling moment/ $(1/2\rho V^2 Sb)$
$C_{l_{ref}}$ = rolling moment induced by potential vortex
$S$ = wing area
$V$ = velocity of the aircraft
$W$ = aircraft weight (lb)
$Y$ = lateral displacement of the penetrating aircraft with respect to the trailing vortex system
$Z$ = vertical displacement of the penetrating aircraft with respect to the plane of the trailing vortices
$\theta$ = pitch angle
$\phi$ = roll angle
$\psi$ = yaw angle

*Subscripts*

$g$ = denotes generating aircraft parameters
$p$ = denotes penetrating aircraft parameters

## Introduction

IN the early 1950's, it was recognized that an aircraft encountering a trailing vortex wake could experience large deviations from its flight path. The magnitude of the deviation depends upon the strength of the vortex wake that is encountered.

It can be shown that the strength of the vortex wake is proportional to the weight and inversely proportional to the forward velocity of the generating aircraft. This implies that the strongest wakes are found behind heavy aircraft during take-off or landing. Therefore, the trailing vortex hazard is greatest in the terminal area for the following reasons: the wakes are stronger; the probability of an encounter is more likely due to the increase in air traffic; and the low altitude of aircraft in the terminal area may not be sufficient to execute a recovery from a vortex upset.

During the past decade, there has been a renewed interest in the hazard due to aircraft trailing vortices. This interest was prompted primarily by the introduction of the large wide body jet transports (DC-10, B-747, L-1011) into commercial service. Such aircraft have very strong wakes that can be potentially hazardous to aircraft in the 80,000 lb class. Much of the analytical and experimental research conducted during this period deals with methods of predicting the vortex roll up, the vortex velocity field, and its rate of decay. There are, however, two areas which have received very little attention. These are the effect of atmospheric conditions on the transport and decay of the vortices and the dynamic behavior of an aircraft penetrating the vortex system.[1-5] It is the latter topic which is the subject of this paper.

## Simplified Analysis

Before discussing the results from the digital simulation, it would be instructive to examine the results from a simplified analysis of a vortex encounter. Consider the situation of an aircraft suddenly penetrating along the axis of the vortex core, one could qualitatively assess the vortex hazard by comparing the vortex induced rolling moment to the control moment available due to aileron deflection. Figures 1 and 2, taken from Ref. 1, show the influence of the relative span and lateral displacement on the vortex induced rolling moment coefficient. Figure 1 was obtained using a modified strip theory, whereas Fig. 2 was computed using a lifting surface program. The vortex strength of the generating wing was approximately that which would be generated by a DC-9 during an approach. Figure 1 shows that aircraft having a span of less than approximately two-thirds the generating aircraft could experience vortex induced rolling moments exceeding the aileron control. Figure 2 shows the influence of the lateral displacement on the vortex induced rolling moment. Note that the vortex induced rolling moment reverses sign as the vortex moves toward the wing tip.

Figure 3 shows the influence of increasing the vortex core on the vortex induced rolling moment. In this figure, the rolling moment coefficient has been normalized to the rolling moment coefficient that would be induced on the wing by a potential vortex of equal strength. The curve indicated that a substantial core enlargement with the associated reduction in

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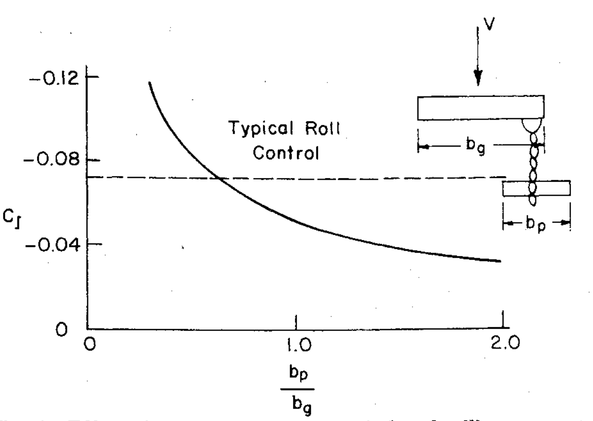

Fig. 1 Effect of relative span on vortex induced rolling moment.

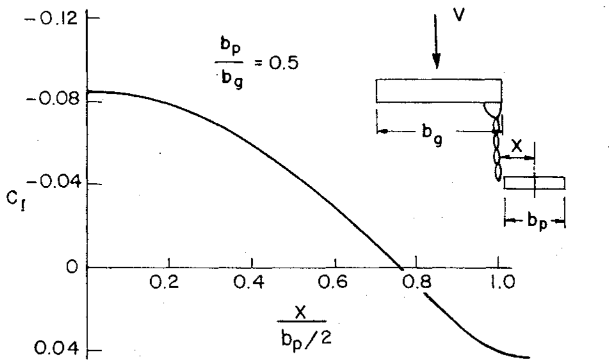

Fig. 2 Effect of lateral displacement on vortex induced rolling moment.

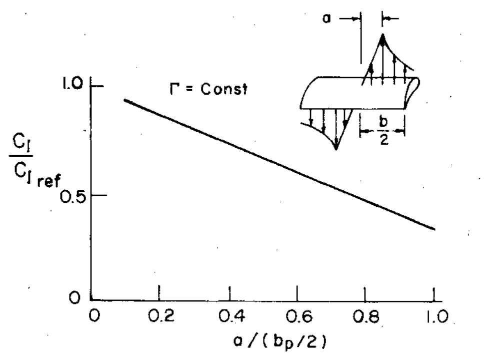

Fig. 3 Effect of vortex core size on vortex induced rolling moment.

peak tangential velocity would be required in order to reduce the vortex induced rolling moment. In order to reduce the rolling moment to half that induced by a potential vortex, the core size would have to be enlarged to approximately 75% of the span of the penetrating aircraft. Thus, devices, that modify the vortex structure by increasing the core radius and reducing the peak velocity may not materially reduce the vortex hazard unless the modification to the vortex causes it to decay faster. A similar conclusion was determined experimentally by El-Ramly.[6].

## Digital Simulation

In this section, a discussion of the dynamic response of a light and medium size aircraft penetrating the vortex system of a larger aircraft will be presented. The responses were calculated by using a digital computer program to solve the equations of motion of an aircraft flying into a vortex wake. A detailed description of the computer program can be found in Ref. 7.

The computer program included the six rigid body equations of motion in a dimensional nonlinearized form. The unsteady forces and moments acting on the aircraft were determined by using a modified strip theory. Admittedly, a more rigorous lifting surface theory method would be preferred, however, the strip theory method was found to yield accurate results when compared to the more exact methods. Thus, for the sake of simplicity and flexibility, the strip theory method was employed. The vortex velocity field created by the lead aircraft was computed from the vortex decay model presented in Ref. 8.

The pilot response to the vortex disturbance was introduced by using the pilot transfer functions presented in Ref. 9. Although the pilot transfer functions were not designed for the large excursions that might occur during a vortex encounter, it is felt that the pilot models would give a qualitative assessment of the pilot's ability to control the upset due to a vortex encounter.

## Penetration Along the Vortex Axis

The most hazardous encounter occurs when the following aircraft penetrates along the axes of the vortex core. Such a penetration is most likely to occur during final approach or shortly after take-off. In either case, the following aircraft can ill afford to have any large deviation from its normal flight path.

For the separation times considered in this study, the normal downward motion of the system would place the vortices well below the flight path of the following aircraft. However, several investigators have shown that under certain atmospheric conditions, the downward motion of the vortex system can be retarded.[10,11]. There is also evidence that as the vortex system approaches an inversion layer, the vortices behave as though they were approaching the ground. Thus, the vortices would cease their downward movement and begin to spread laterally. Another possibility of encountering a vortex wake in the terminal area would be during the execution of a missed approach by the lead aircraft. The wake of the lead aircraft would be laid down at an altitude well above that of the following aircraft and thus, could descent into the flight path of the trailing aircraft.

Now consider some typical results from the six degree-of-freedom computer simulation. First, examine the response of an aircraft where the pilot controls the roll attitude. That is, the mathematical pilot model attempts to maintain a wings level orientation. Figure 4 illustrates the penetration angles under consideration. $\theta_p$ is the angle between the penetrating aircraft's flight path and the plane of the vortices, whereas $\psi_p$ is defined as the angle between the flight path and the vortex axis when viewed from above. For the cases considered here, the separation distance was 28,000 ft, or approximately 2 min. behind the generating aircraft. Also, it should be pointed out that the flight path of the penetrating aircraft was selected so that it would intersect the vortex core if the aircraft were not influenced by the vortex system. The characteristics of the aircraft are shown in Table 1.

For an aircraft descending into the vortex system at the angle $\Theta_p = -3°$, Figs. 5 and 6 illustrate the effect of the oblique penetration angle $\psi_p$ on the response of the penetrating aircraft. Figure 5 shows the vertical and lateral displacement as well as the roll attitude of a business aircraft penetrating the wake of a light jet transport. In the figures

**Table 1 Aircraft characteristics**

| Aircraft parameters | Business aircraft | Light jet transport (DC-9) | Convair 990 |
|---|---|---|---|
| Wing area, ft$^2$ | 350 | 980 | 2,250 |
| Span, ft | 55 | 90 | 118 |
| Weight, lb | 15,000 | 70,000 | 153,000 |
| Velocity, fps | 190 | 232 | 245 |

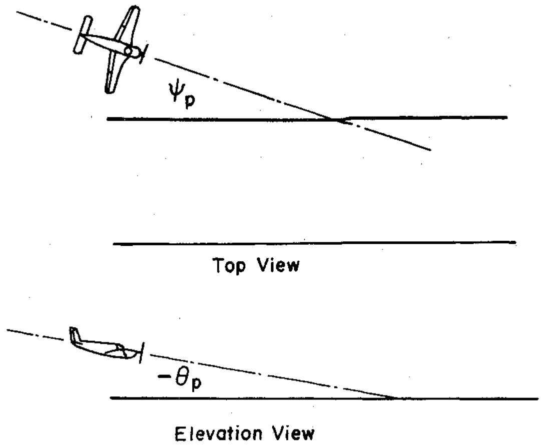


Fig. 4 Sketch of an aircraft penetrating a vortex system.

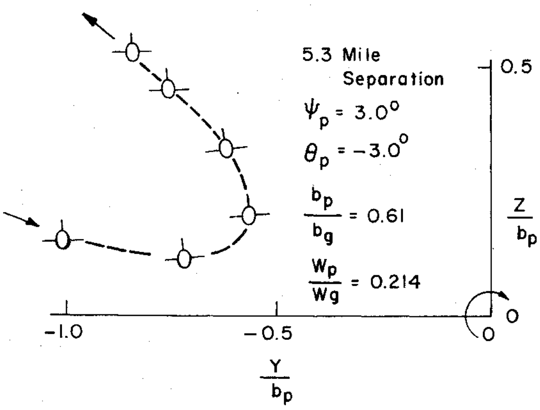


Fig. 5 Business aircraft descending into the wake of a light jet transport ($\psi_p = 3.0°, \theta_p = -3.0°$).

showing the aircraft silhouette, the aircraft is flying into the plane of the sketch. Notice that the aircraft descending into the vortex with $\psi_p = 3°$ rolls away from the vortex system. The aircraft climbs since it is flying into the upwash region of the left vortex. Upon increasing the penetration angle to $\psi_p = 6°$ we see a significant difference in the aircraft's response. Figure 6a shows the aircraft rolling sharply to the right, and at the same time its rate of descent is increased as the aircraft passes into the strong downwash region between the vortices. Figure 6b illustrates the effect of including the pilot in the simulation. Without pilot control, the aircraft rolls to an angle greater than 100° whereas with control input, it rolls approximately 50° before roll recovery takes place. The time history plot of the yaw angle reveals the influence the vertical tail plays in the aircraft's dynamic behavior. As the aircraft enters the vortex field, it is seen to roll in a counter-clockwise direction. This can be explained by referring to Fig. 2 which illustrates the influence of lateral displacement on the induced rolling moment coefficient. Note that the induced rolling moment coefficient reverses sign as the vortex moves toward the wing tip. Thus, as the aircraft approaches the left vortex, it initially rolls in the opposite direction with respect to the vortex field. The pilot's reaction to the disturbance would be to apply aileron control to roll the aircraft in a clockwise direction. Thus, we see that the pilot's initial reaction would be momentarily out of phase with the roll disturbance. Also, during the vortex encounter, the vertical tail experiences an induced velocity from the left causing the aircraft to yaw to the left by approximately 12°. As the aircraft passes into the field of influence of the right vortex, the upsetting roll moment tends to aid the pilot in regaining a wings level attitude. The yaw excursion to the right is due primarily to the roll orientation.

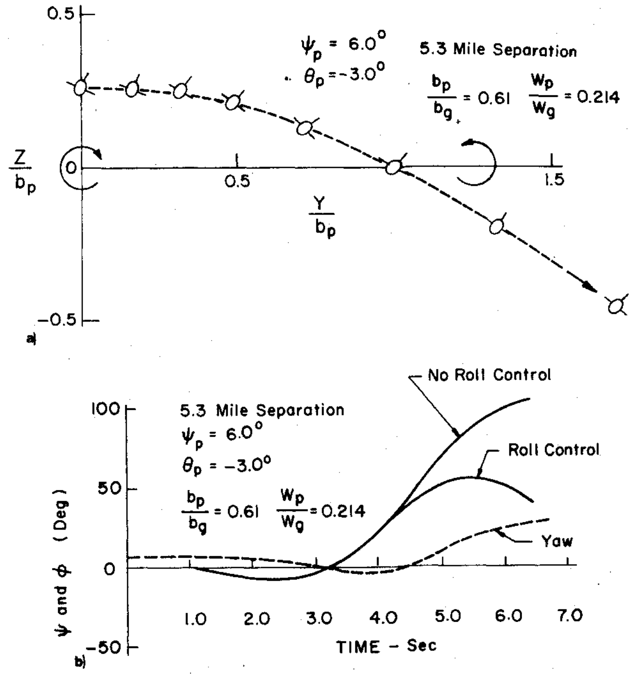


Fig. 6 Business aircraft descending into the wake of a light jet transport ($\psi_p = 6.0°$). a) Vertical and lateral displacement of the penetrating aircraft. b) Time history of roll and yaw angles.

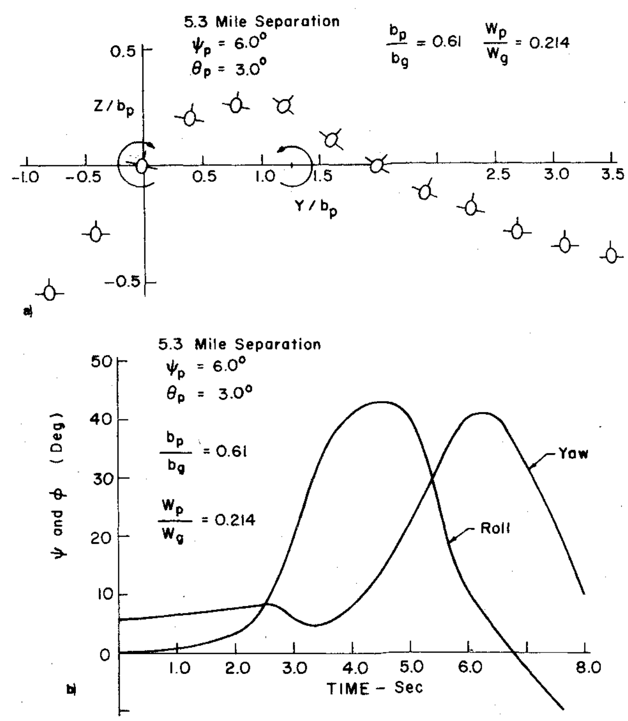


Fig. 7 Business aircraft climbing into the wake of a light jet transport ($\psi_p = 6.0°$, $\theta_p = 3.0°$). a) Vertical and lateral displacement of penetrating aircraft. b) Time history of roll and yaw angles.

Figures 7 and 8 shows the effect of varying the angle $\theta_p$. For these calculations $\psi_p$, is held constant. Here, the aircraft is climbing into the vortex system. The angle $\theta_p$ was varied from 3° to 6°. These figures again indicate the influence of the vertical tail on the aircraft's response. As the aircraft climbs into the velocity field of the left vortex, the vertical tail experiences a side velocity from the right. This gives rise to a positive yawing moment which yaws the airplane to the right. As it passes through the plane of the vortices, its rate of climb diminishes and the aircraft rolls in a clockwise direction. The aircraft in Fig. 7 is initially located approximately 30 ft below

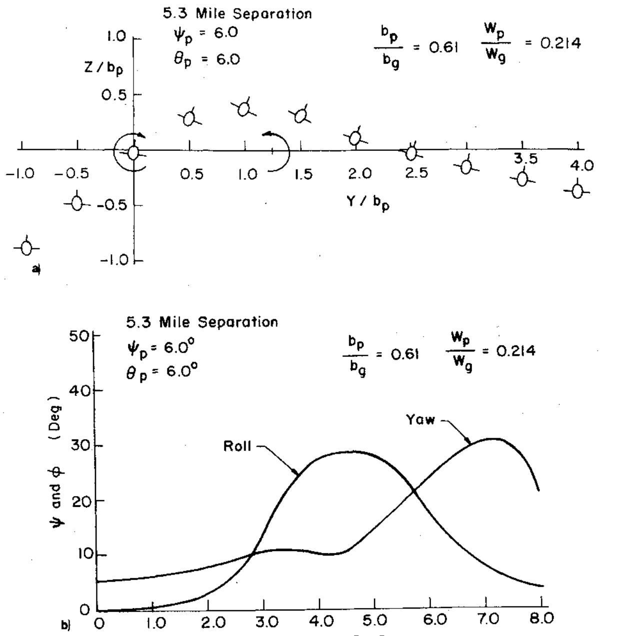


Fig. 8 Business aircraft climbing into the wake of a light jet transport ($\psi_p = 6.0°$). a) Vertical and lateral displacement of penetrating aircraft. b) Time history of roll and yaw angles.

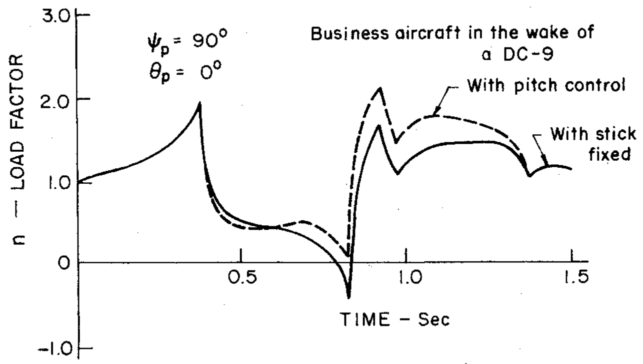


Fig. 9 Normal load factor vs time (perpendicular penetration).

and 41 ft to the left of the left vortex. However, after eight sec, the aircraft is located 22 ft beneath the vortex plane. If the airplane had not encountered the vortex system, it would have been approximately 100 ft above the plane of the vortices. Figure 8 shows a similar result for $\theta_p = 6°$. Based on the results presented here, several conclusions can be made concerning the dynamic behavior of the following aircraft. The calculations showed that even with pilot control, large excursions in roll, yaw, and altitude were experienced by the penetrating aircraft. The results also show the significant influence the vertical tail makes to the aircraft's dynamic response.

## Penetration Perpendicular to the Vortex Axis

Again, this type of penetration is most likely to occur in the terminal area where aircraft are following different traffic patterns. The results shown in Fig. 9 are representative of the cases studied. The normal load factor without control input compares favorably with results obtained in previous investigations.[13] However, when pilot control was added, the normal load factor was increased only slightly. McGowan's calculations, on the other hand, indicated that with elevator input the induced loading could exceed the design limit and, in some cases, the design ultimate load factors. But the results from this study indicated that structural failure was not a serious threat to the penetrating aircraft.

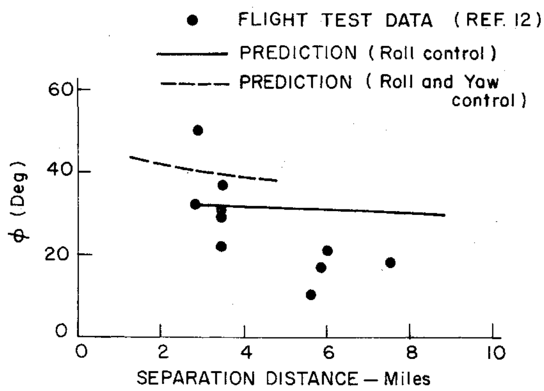


Fig. 10 Maximum roll angle of a DC-9 in the wake of a Convair 990.

## Comparison with Flight Test Data

In an effort to assess the validity of the computer simulation, the results were compared with flight test data obtained from Ref. 12. These data were obtained by the NASA Flight Research Center at Edwards Air Force Base. The probe aircraft were positioned by radar to a specified distance behind the generating aircraft. The aircraft were flown into the vortex wake for two or three min. in order to gather the response data.

The computed results for a DC-9 penetrating the wake of a Convair 990 are presented in Fig. 10 along with the flight test data. Two calculated curves are shown in this figure. The solid curve is based on calculations including only roll control and the dashed curve for both roll and heading control. The calculations using only roll control are low in comparison to the flight test data for separation distances under four miles. However, by maintaing both roll and heading attitude, the prediction is in much closer aggrement with the flight test data. As previously shown, the aircraft is rapidly expelled from the vortex system and therefore by trying to hold his heading, the pilot keeps the aircraft in the influence of the vortices for a longer time resulting in larger roll deviations.

For separation distanced greater than approximately five miles, the simulation over-predicts the maximum roll excursions. This could be attributed to a variety of factors; the most likely are the assumed vortex decay model and the possibility that the vortex system has started to break up. However, in general, the simulation results are representative of an actual vortex encounter.

## Conclusions

Based on the results presented in this paper, the following conclusions have been reached concerning the dynamic behavior of an aircraft penetrating a trailing vortex wake.

1) Reduction of the maximum vortex tangential and core enlargement does not necessarily reduce the vortex hazard.

2) The pilot's reaction could be momentarily out of phase with the upsetting rolling moment.

3) An aircraft descending into a vortex can experience completely different types of response depending upon the penetration angle $\psi_p$. For very shallow angles, the aircraft is rolled away from the vortex system. As $\psi_p$ increases, the aircraft is rapidly rolled to very large roll angles.

4) Aircraft climbing into the vortex system will experience large roll, yaw, and altitude excursions.

The pilot can increase the roll upset by trying to maintain his heading. In so doing, the aircraft remains in the vortex field for a longer time resulting in a larger roll deviation.

6) In the transverse penetration, the pilot's control input did not appreciably increase the load factor as shown in previous studies. For the aircraft used in this study, the results indicate that structural failure is not a serious threat to the penetrating aircraft.

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